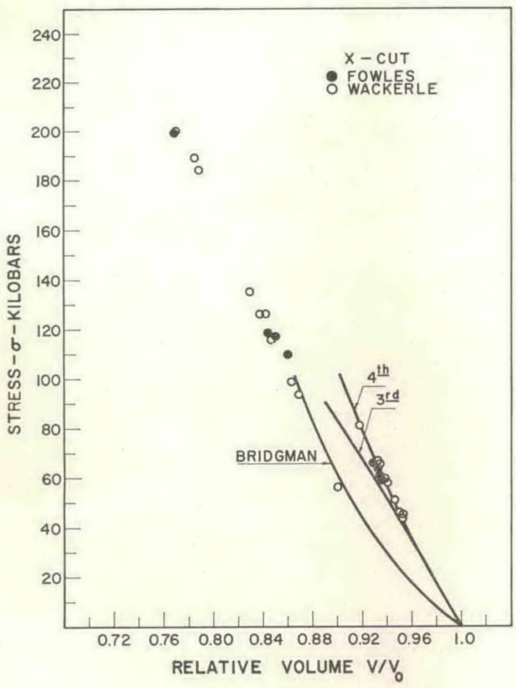

Fig. 7. Stress-volume states resulting from shock compression of X-cut quartz. Solid curve is Bridgman's hydrostatic data. Curves labeled 3rd, 4th are fits based on zero-pressure elastic constants to third- and fourth-order.

In these equations, V is specific volume, u is particle velocity, U is shock velocity, σ is stress normal to the shock front, and ρ is density. Subscripts 0 refer to the state ahead of the shock; subscripts I refer to the state behind the shock. Velocities are with respect to laboratory coordinates. Figures 7, 8, and 9 show the results in the stress specific-volume plane for X, Y, and Z crystals, respectively. *Bridgman*'s [1947] hydrostatic curve based on measurements to 98 kb is shown for comparison. The curves labeled 3rd, 4th, are fits based on low-pressure acoustic measurements and finite elastic strain theory.

These plots show clearly the following important features of the compression: (1) extremely high-amplitude elastic waves, up to 150 kb in Z-cut quartz, (2) loss of rigidity above the elastic limit, as shown by the agreement of the higher-pressure shock data with extrapolation of the hydrostatic data, and (3) lack of a unique value for the Hugoniot elastic limit.

This behavior implies that yielding is not due to dislocation motion, as in a metal, but is analogous (or identical) to fracture. It is shown below that the shear stresses behind the elastic shocks approach the theoretical shear strength of the crystal lattice.

The range of the present data is not sufficient to show clearly the transformation to stishovite, as indicated by Wackerle's higher-pressure measurements.

Finite Elastic Strain Theory

Because the strains behind the elastic shocks are relatively large, it is of interest to examine the agreement of the data with predictions of finite strain theory. Predictions are made possible by the work of *Thurston et al.* [1966] and *McSkimin et al.* [1965] on the third-order elastic constants of quartz. Such comparisons should indicate the extent to which third-order constants are sufficient to describe the stress-strain behavior at strains of the order of 5–10%. The constants are determined from precise acoustic measurements at strains of less

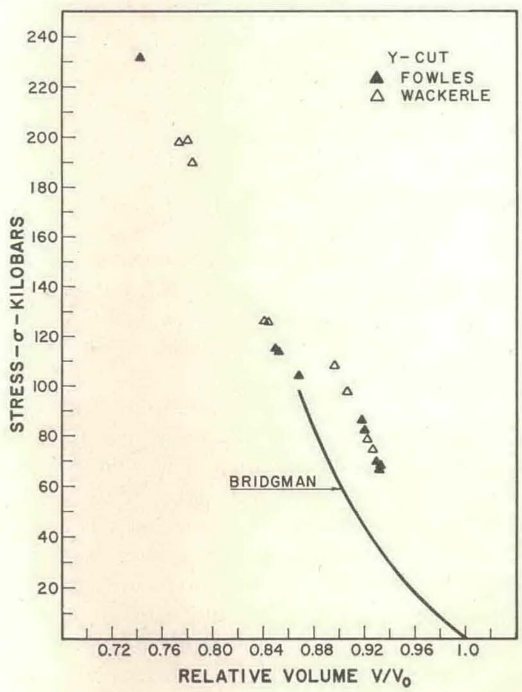

Fig. 8. Stress-volume states resulting from shock compression of Y-cut quartz. Solid curve is Bridgman's hydrostatic data.

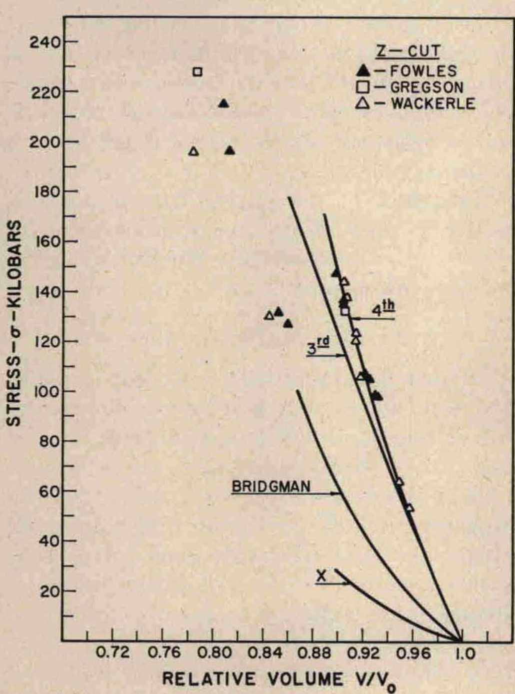

Fig. 9. Stress-volume states resulting from shock compression of Z-cut quartz. Solid curve is Bridgman's hydrostatic data. Dashed curves labeled 3rd, 4th, are fits based on zero-pressure elastic constants to third- and fourth-order. The curve labeled X represents the tangential stresses, calculated from constants up to and including third order.

than 0.1%. *Anderson* [1966] has already shown that the second- and third-order constants alone give reasonably good predictions for hydrostatic compressions of up to about 15% in quartz, provided that the constants are used in the Murnaghan logarithmic equation or the Birch equation of state.

Discrepancies between the observed and predicted stress-strain curves can be used alternatively to evaluate fourth- and higher-order constants or to guide the formulating of improved or more convenient constitutive assumptions. Finally, to the extent that the third-order constants give adequate predictions, the normal stresses parallel to the shock fronts can be calculated from the observed stresses normal to the fronts; hence, the shear stresses sustained (momentarily) by the crystal can be deduced.

Finite strain fundamentals. (This section is a summary of parts of the theory presented by *Thurston* [1964].) Denote the coordinates of a mass element in an initial (unstrained) coordinate system by a_i, and the coordinates in a final (strained) system by x_i, with the transformation given by

$$x_i = x_i(t, a_1, a_2, a_3) \qquad i = 1, 2, 3 \tag{4}$$

where

$$a_i = x_i(t_0, a_1, a_2, a_3)$$

t_0 being a reference time. The x_i are thus Eulerian, or spatial, coordinates and the a_i Lagrangian, or material, coordinates.

For this transformation one can derive an expression for the ratio of specific volumes

$$V/V_0 = J = |\partial x_i/\partial a_s| \tag{5}$$

J is thus the determinant of the Jacobian of the transformation or the 'functional determinant.'

The strain N_{jk} is defined, somewhat arbitrarily, from the difference in the squares of the lengths of line elements by

$$2N_{jk}\, da_j\, da_k = dx_i\, dx_i - da_i\, da_i$$
$$N_{jk} = \frac{1}{2}\left(\frac{\partial x_i}{\partial a_j}\frac{\partial x_i}{\partial a_k} - \delta_{jk}\right) \tag{6}$$

Here and in the following the Einstein summation convention for repeated subscripts applies. δ_{jk} is the Kronecker delta.

Expanding the internal (strain) energy in a power series in the strains, one obtains (at constant entropy)

$$\rho_0[E(N, S) - E(0, S)]$$
$$= (1/2)c_{ijkl}{}^s N_{ij}N_{kl} + (1/6)c_{ijklmn}N_{ij}N_{kl}N_{mn}$$
$$+ (1/24)c_{ijklmnpq}{}^s N_{ij}N_{kl}N_{mn}N_{pq} + \cdots \tag{7}$$

In this expression the $c_{ijk}{}^s \cdots$, represent the second- and higher-order isentropic elastic stiffness coefficients. The first-order term is missing because the reference state is considered to be zero stress and strain.

We now define quantities, called thermodynamic tensions, by

$$t_{ij} = \rho_0(\partial E/\partial N_{ij})_s \tag{8}$$

In terms of these quantities the elastic constants are